

Exercise 53

(a) If the symbol $\llbracket x \rrbracket$ denotes the greatest integer function defined in Example 10, evaluate

$$(i) \lim_{x \rightarrow -2^+} \llbracket x \rrbracket \quad (ii) \lim_{x \rightarrow -2} \llbracket x \rrbracket \quad (iii) \lim_{x \rightarrow -2.4} \llbracket x \rrbracket$$

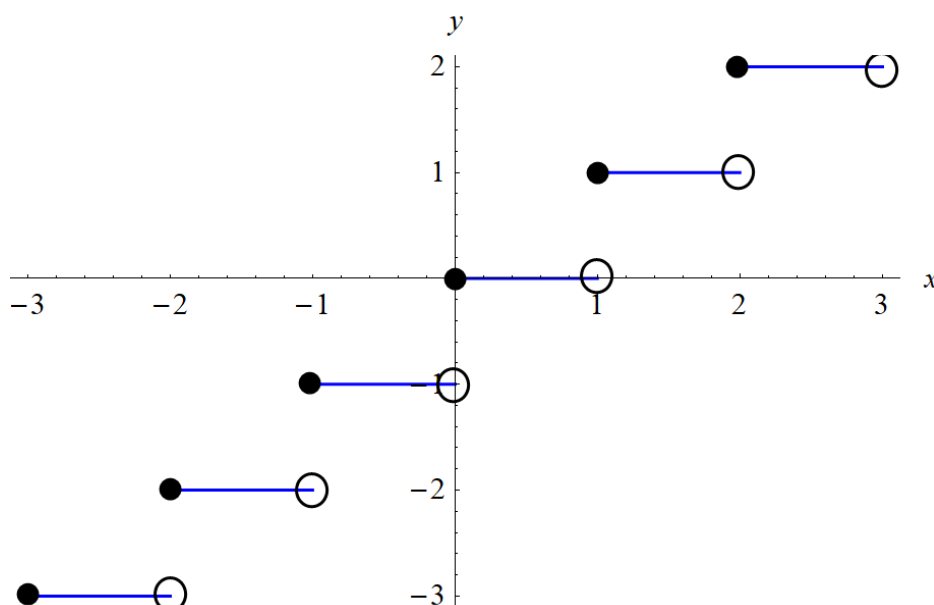
(b) If n is an integer, evaluate

$$(i) \lim_{x \rightarrow n^-} \llbracket x \rrbracket \quad (ii) \lim_{x \rightarrow n^+} \llbracket x \rrbracket$$

(c) For what values of a does $\lim_{x \rightarrow a} \llbracket x \rrbracket$ exist?

Solution

Below is a graph of the greatest integer function.



Use it to evaluate the limits in question.

$$\lim_{x \rightarrow -2^+} \llbracket x \rrbracket = -2$$

$$\lim_{x \rightarrow -2} \llbracket x \rrbracket \text{ does not exist because } \lim_{x \rightarrow -2^-} \llbracket x \rrbracket \neq \lim_{x \rightarrow -2^+} \llbracket x \rrbracket$$

$$\lim_{x \rightarrow -2.4} \llbracket x \rrbracket = -3$$

$$\lim_{x \rightarrow n^-} \llbracket x \rrbracket = n - 1$$

$$\lim_{x \rightarrow n^+} \llbracket x \rrbracket = n$$

$\lim_{x \rightarrow a} \llbracket x \rrbracket$ exists whenever $a \neq n$.